UDC 62-1

Muhammad Bello Garba, Ph.D.
Department of Technical Education
Shehu Shagari University of Education, Sokoto, Nigeria

INFLUENCE OF CENTRIFUGAL FORCES ON STRENGTH OF BELT DRIVE PULLEY RIM
ВЛИЯНИЕ ЦЕНТРОБЕЖНЫХ СИЛ НА ПРОЧНОСТЬ КОЛЕСА ШКИВА РЕМЕННОЙ ПЕРЕДАЧИ

Abstract: The paper considers the essence of calculating the strength of a belt drive pulley under the action of centrifugal forces, and conditions are obtained that ensure the strength of the pulley rim.

Keywords: belt drive, strength, pulley rim, centrifugal force, tension

Аннотация: В статье рассматривается сущность расчета прочности шкива ременной передачи под действием центробежных сил, и получены условия, обеспечивающие прочность обода шкива.

Ключевые слова: ременная передача, прочность, обод шкива, центробежная сила, натяжение.

Introduction

High-speed belt drive pulleys are essential drive components for various mechanisms in tractors, automobiles, agricultural machinery, metalworking machines, turbines, fans, and more. Pulley strength depends on the intended use, operating parameters, and characteristics of the machines, their operating conditions, and their service life. At high rotational speeds, plastic deformation or failure may occur in the pulley rim material, which can lead to serious machine failures. The main loads on the belt drive pulley rim are centrifugal forces, which stretch it.

Belt drive systems remain a fundamental means of power transmission in mechanical engineering applications due to their simplicity, flexibility, and cost-effectiveness. However, the performance and durability of such systems are strongly influenced by the stresses developed within the belt and pulley, particularly under high-speed operating conditions. One of the most critical factors affecting these stresses is centrifugal force, which significantly alters belt tension and consequently impacts pulley rim strength.

According to Shigley's Mechanical Engineering Design, the total tension in a moving belt consists of both the effective driving tension and the centrifugal tension, the latter arising due to the mass of the belt moving at high velocity. The centrifugal tension is expressed as a function of belt mass and velocity, and it increases quadratically with speed. This additional tension raises the overall stress within the belt, thereby increasing the load transmitted to the pulley structure [1].

Similarly, classical analyses presented in Theory of Machines show that centrifugal tension reduces the effective frictional grip between the belt and pulley by acting outward from the centre of rotation. This reduction in normal reaction between the belt and pulley necessitates higher initial tension to maintain power transmission, which in turn increases the net force acting on the pulley rim [2]. This condition is particularly critical when operating near the maximum power transmission point, where centrifugal effects become dominant.

Further insights into pulley force mechanics are provided by Theory of Machines, where it is emphasized that the equilibrium of forces on a pulley must account for both tight-side and slack-side tensions, along with centrifugal contributions. The combined effect of these forces leads to increased radial and tangential stresses on the pulley rim, which may induce deformation or fatigue failure if not properly accounted for in design [3].

From a design perspective, Design of Machine Elements highlights that pulley rims are subjected to tensile (hoop) stress due to belt tension, in addition to bending stresses caused by uneven load distribution. The presence of centrifugal tension amplifies these stresses, necessitating increased rim thickness and careful material selection to ensure structural integrity under dynamic conditions [4].

Analytical and theoretical studies also support these observations. For instance, [5] demonstrated that centrifugal force reduces the effective contact pressure between the belt and pulley, thereby altering load distribution and increasing the likelihood of slip. This change in force transmission dynamics indirectly contributes to higher stress concentrations in the pulley rim. Likewise, [6,7] provided a rigorous treatment of force and torque transmission in pulley systems, showing how tension variations along the belt contribute to complex loading conditions on the pulley.

In summary, the literature consistently shows that centrifugal forces play a significant role in modifying belt tension, reducing frictional grip, and increasing the overall load transmitted to pulley rims. These effects necessitate careful consideration in pulley design, particularly in high-speed applications where centrifugal tension becomes substantial. Failure to account for these forces can lead to excessive stress, material fatigue, and eventual structural failure of the pulley system.

The objective of the paper is to ascertain which places on the belt drive rim are to be checked for strength under the action of centrifugal forces.

Materials and Methods

The pulley rotates at a constant angular velocity ω (Figure 1). The forces acting on the disk are assumed to be symmetrical relative to the axis and uniformly distributed across the width of the pulley rim.

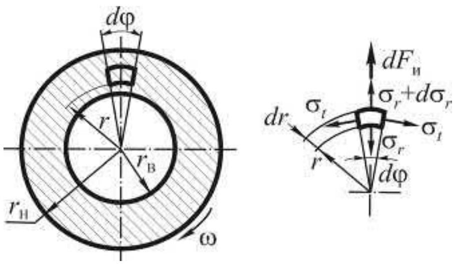

Figure 1 – Forces acting of pulley disk (rim)

The calculation is based on the following hypotheses:

1. the pulley rim material is subject to plane stress, i.e., normal stresses in planes perpendicular to the axis are zero.
2. the radial and circumferential normal stresses depend only on the current radius *r*.

Let us isolate the rim element from the pulley using two radial planes and two cylindrical surfaces and examine its equilibrium (figure). The element is subject to forces due to normal radial and circumferential stresses, as well as the inertial force dF, which, as is known, is equal to the product of the element's mass and the normal acceleration.

$$dF_i = \frac{\gamma}{g} rhd\varphi \omega^2 rdr \quad (1)$$

Where,

γ is the specific gravity of the material.

g is the acceleration due to gravity.

r is the current radius, the distance from the point under consideration to the pulley's axis of rotation.

h is the pulley rim width.

The force dFi is directed radially away from the axis of rotation.

To formulate the equilibrium equation for the selected element, we project all forces onto a radius coinciding with the bisector of angle dω:

$$(\sigma_r + d\sigma_r(r + dr)hd\varphi - \sigma_r rhd\varphi - 2\sigma_t hdrsin\left(\frac{d\varphi}{2}\right) + \frac{\gamma}{g} rhd\varphi \omega^2 rdr = 0 \quad (2)$$

Where σ_r, σ_t – normal (radial and circumferential) stresses respectively.

After transformations, equation (2) takes the following form:

$$\frac{d}{dr}(\sigma_r r) - \sigma_t = -\frac{\gamma}{g} \varphi^2 \omega^2 = 0 \quad (3)$$

Relative linear deformations in the radial and circumferential directions through the radial displacement *u* and the generalized Hooke's law can be represented as

$$\varepsilon_r = \frac{du}{dr} = \frac{1}{E}(\sigma_r - \mu\sigma_t) \quad (4)$$
$$\varepsilon_t = \frac{u}{r} = \frac{1}{E}(\sigma_t - \mu\sigma_r) \quad (5)$$

Where E and μ are the elastic modulus and Poisson's ratio of the pulley material, which are physical constants.

From equations (4) and (5), we express the normal radial and circumferential stresses

$$\sigma_r = \frac{E}{1-\mu^2}\left(\frac{du}{dr} + \mu\frac{u}{r}\right) \quad (6)$$
$$\sigma_t = \frac{E}{1-\mu^2}\left(\frac{u}{r} + \mu\frac{du}{dr}\right) \quad (7)$$

Substituting (6) and (7) into the equilibrium equation, we obtain a differential equation for the radial displacements of the points of the rotating pulley rim.

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} + \frac{1-\mu^2}{E}\frac{\gamma}{g}\omega^2 r^2 \quad (8)$$

The solution to equation (8) is represented as:

$$u = Cr + \frac{D}{r} - \frac{1-\mu^2}{8E}\cdot\frac{\gamma}{g}\omega^2 r^3 \quad (9)$$

Where C and D are constants of integration.

Substituting (9) into equations (6) and (7), we obtain formulas for radial and circumferential stresses.

$$\sigma_r = \frac{E}{1-\mu^2}\left(C(1-\mu) - D(1-\mu)\frac{1}{r^2}\right) - \frac{\gamma}{8g}\omega^2(3+\mu)r^2 \quad (10)$$
$$\sigma_t = \frac{E}{1-\mu^2}\left(C(1-\mu) + D(1-\mu)\frac{1}{r^2}\right) - \frac{\gamma}{8g}\omega^2(1+3\mu)r^2 \quad (11)$$

The integration constants are found by solving a system of two equations obtained by substituting into expression (10) the initial conditions for σ_r on the outer and inner surfaces of the pulley rim when: $r = r_H, \sigma_r = 0, r = r_B, \ \sigma_r = 0$,

$$C = \frac{\gamma\omega^2(1-\mu)(3+\mu)(r_H^2 + r_B^2)}{8gE} \quad (12)$$
$$D = \frac{\gamma\omega^2(1+\mu)(3+\mu)(r_H^2 r_B^2)}{8gE} \quad (13)$$

By substituting the constants of integration into expressions (10) and (11), we obtain the formulas for determining radial and circumferential stresses as

$$\sigma_r = \left(r_H^2 + r_B^2 - \frac{r_H^2 r_B^2}{r^2} - r^2\right)\cdot\frac{\gamma}{8g}\omega^2(3+\mu) \quad (14)$$
$$\sigma_t = \left(r_H^2 + r_B^2 - \frac{r_H^2 r_B^2}{r^2} - \frac{1+3\mu}{3+\mu}r^2\right)\cdot\frac{\gamma}{8g}\omega^2(3+\mu) \quad (15)$$

From expression (15) above, it is evident that σ_t takes the greatest values on the inner surface of the pulley rim, where σ_r is equal to zero. Then the strength of the pulley rim at points on its inner surface will be ensured when the following condition is fulfilled

$$\sigma_{max} = \sigma_t = \left(2r_H^2 + r_B^2\left(1 - \frac{1+3\mu}{3+\mu}r^2\right)\right) \cdot \frac{\gamma}{8g}\omega^2(3+\mu) \le [\sigma] \quad (16)$$

Where $[\sigma]$ is the permissible stress.

It is also necessary to check the strength at the points in the rim cross-section where the greatest σ_r occurs. The position of these points, determined by the radius r_m , is determined by equating the expression for the first derivative to zero: $\frac{d\sigma_r}{dr} = 0$.

As a result, we obtain $r_m = \sqrt{r_H r_B}$

Then the stresses at the points of the rim cross-section located at a distance r_m from the axis will be equal to

$$\sigma_{rmax} = (r_H^2 + r_B^2) \cdot \frac{\gamma}{8g}\omega^2(3+\mu) = \sigma_1 \quad (17)$$

$$\sigma_t = \left(r_H^2 + r_B^2 - \frac{1+3\mu}{3+\mu}r_H r_B\right) \cdot \frac{\gamma}{8g}\omega^2(3+\mu) = \sigma_2 \quad (18)$$

Where σ_1 , σ_2 - are the principal (main) stresses

The points on the rim under consideration are subject to plane stress, since the principal stress is $\sigma_3 = 0$. To assess the strength Energy Theory is used:

$$\sigma_{eq.IV} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \le [\sigma] \quad (19)$$

Conclusion

Thus, the strength of a belt drive pulley rim under the action of centrifugal forces must be checked at the cross-section points located on the inner surface according to condition (16), and at the cross-section points at a distance of $r_m = \sqrt{r_H r_B}$ by fulfilling condition (19).

References

1. Budynas, R.G., & Nisbett, J. K. (2020). *Shigley's Mechanical Engineering Design (11th Ed.)*. McGraw-Hill Education.

2. Khurmi, R. S., & Gupta, J. K. (2019). *Theory of Machines (14th Ed)*. S. Chand Publishing.

3. Rattan, S. S. (2014). *Theory of Machines (4th Ed.)*. McGraw-Hill Education.

4. Bhandari, V. B. (2010). *Design of Machine Elements (3rd ed)*. Tata McGraw Hill.

5. Firbank, T. C. (1970). Mechanics of Belt Drives. *Journal of Mechanical Engineering Science, 12(3)*, 220-230.

6. De Oliveira P. M.C. & Lemons, N.A. (2017). *Forces and Torque of a String on a Pulley*. arXiv.https://arxiv.org/abs/1706.10139.

7. Podskrebko, M. D. (2009). *Strength of Materials. Fundamentals of the Theory of Elasticity, Plasticity, Creep and Fracture Mechanics*: Minsk, Higher School, p 670.