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WAVE PACKETS IN AXIALLY COMPRESSED CYLINDRICAL SHELL

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Abstract: The problem of propagation of bending waves in the axially compressed non-circular cylindrical shell is analysed. On the edges that may be not plane the conditions of a joint support are considered. The axial load is a function of circumferential co-ordinate. Initial displacement are localised near some meridian. Solution is constructed as a superposition of the localised families (packets) of bending waves. By using complex WKB-method the problem is reduced to the sequence of one-dimensional initial boundary value problem.

1. Introduction

Consider an elastic non-circular cylindrical shell that is sufficiently thin for the applicability of both the assumptions of the classical shell theory and the asymptotic methods. The orthogonal co-ordinate system (x, φ) is assumed as shown in Figure1 so that the first quadratic form of the middle surface has the form $R^2(ds^2 + d\varphi^2)$. The shell edges may be not plane so that $s_1(\varphi) \le s \le s_2(\varphi)$, where $s = x/R$ is a dimensionless longitudinal co-ordinate, R is the characteristic size of the middle surface, φ is a circumferential co-ordinate $(\varphi_1 \le \varphi \le \varphi_2)$. In this case the variable curvature radius $R_2 = R/k(\varphi)$. The thickness h, Young's modulus E, Poisson's ratio ν and the mass density ρ may be functions of φ. The case when the shell experiences the axial non-uniform (in the circumferential direction) and non-stationary load $T_1^*(\varphi, t)$ is considered here.

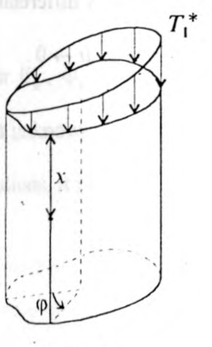

Figure 1

We will study non-stationary localised wave processes being accompanied by the formation of a large number of short waves running in the circumferential direction. It is assumed also that $T_1^*(\varphi, t)$ is a slowly varying function so that the dynamic stress state of the shell due to the axial stress T_1^* may be considered as the membrane one. Taking into account these assumptions, for the analysis of running waves the semi-membrane shell equations [1, 2]

$$\begin{gathered}\mu^2\Delta\left(d(\varphi)\Delta W\right)+\dot{T}_1(\varphi,t)\frac{\partial^2 W}{\partial s^2}-k(\varphi)\frac{\partial^2\Phi}{\partial s^2}+m(\varphi)\frac{\partial^2 W}{\partial t^2}=0,\\ \mu^2\Delta\left(g^{-1}(\varphi)\Delta\Phi\right)+k(\varphi)\frac{\partial^2 W}{\partial s^2}=0\end{gathered} \tag{1}$$

written in the dimensionless form may be used. Here

$$\Delta=\frac{\partial^2}{\partial s^2}+\frac{\partial^2}{\partial\varphi^2},\quad \mu^4=\frac{h_0^2}{12\left(1-\nu_0^2\right)R^2},\quad W=\mu^2\frac{W^*}{R},\quad \Phi=\frac{\mu^{-2}\Phi^*}{h_0 E},$$

$$T_1^*=-E_0h_0\mu^2T_1,\quad d=\frac{Eh^3\left(1-\nu_0^2\right)}{E_0h_0^3\left(1-\nu^2\right)},\quad g=\frac{Eh}{E_0h_0},\quad m=\frac{\rho h}{\rho_0h_0},\quad t=t^*/t_c,\quad t_c^2=\frac{R^2\rho_0}{E_0\mu^2}, \tag{2}$$

where W^*, Φ^* are the normal displacements and stress function, respectively, t^* is time, t_c is the characteristic time, $0<\mu$ is a small parameter, h_0, E_0, ν_0, ρ_0 are the characteristic values of h, E, ν, ρ (for example, at $\varphi=0$), respectively.

All the functions $s_j(\varphi), d(\varphi), g(\varphi), T_1(\varphi, t), k(\varphi), m(\varphi)$ are assumed to be infinitely differentiable and together with the derivatives with respect to φ be quantities of the order $O(1)$ at $\mu\to 0$.

The shell edges $s=s_1(\varphi), s=s_2(\varphi)$ are supposed to satisfy Navier's conditions

$$W=\frac{\partial^2 W}{\partial s^2}=0,\quad \Phi=\frac{\partial^2\Phi}{\partial s^2}=0. \tag{3}$$

The wave forms of motion caused by the initial displacements and velocities

$$W\big|_{t=0} = \widetilde{W}_0(s,\varphi,\mu)F_0\,,\quad \dot{W}\big|_{t=0} = \mathrm{i}\mu^{-1}\widetilde{V}_0(s,\varphi;\mu)F_0 \tag{4}$$

where

$$F_0 = F_0(\varphi,\mu) = \exp\left\{\mathrm{i}\mu^{-1}\left(a_0\varphi + \frac{1}{2}b_0\varphi^2\right)\right\},\quad \mathrm{Im}\,b_0 > 0 \tag{5}$$

will be studied below. In equations (5), $a_0 > 0, b_0$ are constants, and $\widetilde{W}_0, \widetilde{V}_0$ are the complex-valued functions such that

$$\frac{\partial^m \widetilde{W}_0}{\partial s^m},\ \frac{\partial^m \widetilde{V}_0}{\partial s^m} \sim \mu^{-m} \text{ at } \mu \to 0,\ m = 0, 1, \ldots$$

The real and imaginary parts of functions (4) with account taken of the last inequality in equations (5), define the two initial wave packets (WP) localised near the generator $\varphi = 0$. They approximate perturbations which may be generated in the shell by some transient forces applied along the line $\varphi = 0$. The similar initial wave formations near the "weakest" line [1] may arise in the shell with variable geometrical and physical characteristics under parametric excitations [3].

2. Method of solution

As the initial conditions (5) represent the WP with centre at the line $\varphi = 0$, it is natural to seek a solution of linear system (1) in the form of superimposing wave packets travelling in the circumferential direction [4]

$$W = \sum_{n=1}^{N} W_n\,,\quad \Phi = \sum_{n=1}^{N} \Phi_n\,. \tag{6}$$

The pair W_n, Φ_n will be called the nth WP. Let $\varphi = q_n(\varphi)$ be the packet centre of the nth WP, where $q_n(t)$ is unknown twice differentiable function such that $q_n(0) = 0$. In view of the local character of the solutions, it is convenient to introduce a local co-ordinate system connected with the centre $q_n(t)$:

$$\varphi = q_n(t) + \mu^{1/2}\xi_n\,. \tag{7}$$

Assuming that the waves have large variability in the axial direction s, we will scale:

$$s = \mu\varsigma . \tag{8}$$

Following to [4, 5], the functions W_n, Φ_n are assumed to be of the form

$$W_n = \sum_{m=0}^{\infty} \mu^{m/2} w_{nm}(\varsigma, \xi_n, t) F_n, \quad \Phi_n = \sum_{m=0}^{\infty} \mu^{m/2} \chi_{nm}(\varsigma, \xi_n, t) F_n, \tag{9}$$

$$F_n = \exp\left\{ \mathrm{i}\left[\mu^{-1} \int_0^t \omega_n(\tau) d\tau + \mu^{-1/2} p_n(t)\xi_n + \frac{1}{2} b_n(t)\xi_n^2 \right] \right\},$$

where $\omega_n(t)$, $p_n(t)$, $b_n(t)$ are twice differentiable functions, $\operatorname{Im} b_n(t) > 0$ for any $t > 0$, and w_{nm}, φ_{nm} are polynomials in ξ_n. All the coefficients in equations (1) are expanded into the series in $\mu^{1/2}\xi_n$ in a neighbourhood of the centre $\varphi = q_n(t)$.

Substituting (6) - (9) into system (1) and boundary conditions (3) produces the sequence of one-dimensional boundary value problems [4, 5]

$$\sum_{j=0}^{m} L_{nj} X_{nm-j} = 0, \quad m = 0, 1, 2, \ldots, \tag{10}$$

$$L_{nj} = \begin{pmatrix} l_{nj11} & l_{nj12} \\ l_{nj21} & l_{nj22} \end{pmatrix}, \quad X_{nm} = (w_{nm}, \chi_{nm})^{\mathrm{T}}$$

with corresponding boundary conditions. Here

$$l_{n011} = d[q_n(t)]\left[\frac{\partial^2}{\partial\varsigma^2} - p_n^2(t)\right]^2 + T_1[q_n(t), t]\frac{\partial^2}{\partial\varsigma^2} + m[q_n(t)][\omega_n(t) - \dot{q}_n(t) p_n(t)]^2,$$

$$l_{n012} = -k[q_n(t)]\frac{\partial^2}{\partial\varsigma^2}, \quad l_{n021} = -l_{n012}, \quad l_{n022} = g^{-1}[q_n(t)]\left[\frac{\partial^2}{\partial\varsigma^2} - p_n^2(t)\right]^2, \tag{11}$$

and the boundary conditions for w_{n0}, χ_{n0} are as follows:

$$w_{n0} = \frac{\partial^2 w_{n0}}{\partial\varsigma^2} = 0, \quad \chi_{n0} = \frac{\partial^2 \chi_{n0}}{\partial\varsigma^2} = 0. \tag{12}$$

Because of awkwardness, the matrix operators L_{nj} as well as the boundary conditions at $j \geq 1$ are not written out here. The symbol T means a transposition.

In the zeroth order approximation (m = 0), one has the homogeneous boundary value problem (10), (12). It has the non-trivial solution

$$X_{n0} = P_{n0}(\xi_n, t)Y_n(t)z_n(\varsigma, q_n(t)) \tag{13}$$

if

$$\omega_n = \dot{q}_n p_n \pm H_n(t, p_n, q_n), \tag{14}$$

where

$$z_n(\varsigma, \varphi) = \sin \lambda_n(\varphi)(\varsigma - \varsigma_1(\varphi)), \quad \lambda_n = \frac{\pi n}{\varsigma_2(\varphi) - \varsigma_1(\varphi)}, \quad \varsigma_l = \mu^{-1} s_l, \ l = 1,2, \quad n = 1, 2, \ldots \tag{15}$$

and

$$H_n = \sqrt{\frac{1}{m(q_n)}\left(d(q_n)(\lambda_n^2(q_n) + p_n^2)^2 - T_1(q_n, t)\lambda_n^2(q_n) + \frac{k^2(q_n)\lambda_n^4(q_n)g(q_n)}{m(q_n)(\lambda_n^2(q_n) + p_n^2)^2}\right)} \tag{16}$$

is Hamiltonian function. The vector $Y_n(t) = (1, y_n(t))^T$ is not written out here.

In the first and second order approximations, one has the non-homogeneous boundary-value problems (10) with corresponding boundary conditions for w_{nm}, χ_{nm} $(m \geq 1)$. The compatibility conditions for these problems yield the Hamiltonian system

$$\dot{q}_n = H_p(t, p_n, q_n), \quad \dot{p}_n = -H_q(t, p_n, q_n), \quad q_n(0) = 0, \quad p_n(0) = a_0, \tag{17}$$

the Riccati equation

$$\dot{b}_n + H_{pp} b_n^2 + 2H_{pq} b_n + H_{qq} = 0, \quad b_n(0) = b_0 \tag{18}$$

and the amplitude equation

$$A_{n2}\frac{\partial^2 P_{n0}}{\partial \xi_n^2} + A_{n1}\xi_n \frac{\partial P_{n0}}{\partial \xi_n} + \left[A_{n0} + \mathrm{i}\frac{\partial}{\partial t}\right] P_{n0} = 0 \tag{19}$$

with respect to polynomial $P_{n0}(\xi_n, t)$. In the last equation, A_{n0}, A_{n1}, A_{n2} are certain functions of time t.

To find vectors X_{nj} for $j \geq 1$, it is necessary to consider the higher approximations.

3. Conclusion

It may be seen that the constructed solution (6), (9) represents the superposition of the nth WP travelling in the circumferential direction. The signs (±) in equation (14) indicate the availability of two branches (positive and negative) of the solution corresponding to the n^+th and n^-th WP running in the opposite directions. Analysis of this solution shows that the behavior of the nth WP depends on both the correlation of the shell parameters and the load non-homogeneity. For example, consider the cylindrical shell with constant geometrical and physical parameters under stationary non-uniform axial forces $T_1(\varphi)$. Let the centre of the initial WP to coincide with the "weakest" line $\varphi = 0$ [1], where $T_1'(0) = 0, T_1''(0) < 0$. Analysis of the Hamiltonian system (17) indicates that if the "energy" of the initial WP is not large then the reflections of the nth WP from some generator $\varphi = \varphi_r$ are possible. Numerical solutions of equations (17) and (18) shows that these reflections may be accompanied by strong focusing and growth of the amplitudes of the nth WP.

4. Acknowledgements

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5. References

1. Tovstik P.E. *Stability of thin shells: Asymptotic methods*, Nauka, Moscow, 1995.
2. Bolotin V.V. *The Dynamic Stability of Elastic Systems*, State Technical and Theoretical Press, Moscow, 1956.
3. Mikhasev G.I. *Free and Parametric Vibrations of Cylindrical Shells under Static and Periodic Axial Loads*, Technishe Mechanik, Band 17, Heft 3, 1997, p.209-216.
4. Mikhasev G.I. *The localised sets of bending waves in a noncircular cylindrical shell with slanted edges*, Prikladnaya Matematika i Mechanika **60**, 1996, p.635-643.
5. Mikhasev G.I. *Travelling wave packets in an infinite thin cylindrical shell under internal pressure*, Journal of Sound and Vibration, **209**, 1998, p.543-559.

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