MAXWELL ELECTRODYNAMICS IN MEDIA, GEOMETRY EFFECTS ON CONSTITUTIVE RELATIONS

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I. INTRODUCTION

The problems of constitutive relations in Maxwell electrodynamics, their possible form, its behavior under the motion of the reference frame, its connection with Special Relativity theory, interplay between constitutive relations and gravity are reviewed. The main accent in our treatment is the known possibility to simulate material media by geometrical methods.

These problems have a long history. We can track interest to the problem in the huge literature that has been produced on this issue. Note that Gordon [2] was first seemed largely interested in trying to describe dielectric media by an effective metrics; Gordon tried to use a gravitational field to mimic a dielectric medium. The idea was taken up and developed by Tamm and Mandel'stam [3, 4]; also see [5-20].

II. GEOMETRICAL MODELING OF THE CONSTITUTIVE RELATIONS IN ELECTRODYNAMICS

The basic relations are as follows. Effective constitutive equation generated by the Riemannian geometry with metric $g_{\alpha\beta}(x)$ have the form

$$D^i = \epsilon_0\, \epsilon^{ik}(x)\, E_k + \epsilon_0 c\, \alpha^{ik}(x)\, B_k\,, \quad H^i = \epsilon_0 c\, \beta^{ik}(x)\, E_k + \mu_0^{-1}\, (\mu^{-1})^{ik}(x)\, B_k\,.$$

Four dimensionless (3×3)-matrices $\epsilon, \alpha, \beta, \mu^{-1}$ are not independent because they are bilinear functions of only 10 components of the symmetrical tensor $g_{\alpha\beta}(x)$:

$$\epsilon^{ik} = \sqrt{-g}(g^{00}g^{ik} - g^{0i}g^{0k}), \quad \alpha^{ik} = +\sqrt{-g}g^{ij}g^{0l}\epsilon_{ljk},$$

$$\beta^{ik} = -\sqrt{-g}g^{0j}\epsilon_{jil}g^{lk}(x), \quad (\mu^{-1})^{ik} = 12\sqrt{-g}\epsilon_{imn}g^{ml}g^{nj}\epsilon_{ljk}(\mu^{-1})^{ik}.$$

These tensors obey the following symmetry constraints: $\epsilon^{ik} = +\epsilon^{kl}$, $(\mu^{-1})^{ik} = +(\mu^{-1})^{ki}$, $\beta^{kl}(x) = \alpha^{ik}$; so the (6×6)-matrix defining constitutive equations is symmetrical. Metrical tensors which are the most interesting in the General relativity have a quasi-diagonal structure $g_{0i}(x) = 0$, and the corresponding constitutive relations simplify

$$\epsilon^{ik} = \sqrt{-g}g^{00}\begin{vmatrix} g^{11} & g^{12} & g^{13} \\ g^{21} & g^{22} & g^{23} \\ g^{31} & g^{32} & g^{33} \end{vmatrix}, \qquad (\mu^{-1})^{ik} = \sqrt{-g}\begin{vmatrix} G^{11} & G^{12} & G^{13} \\ G^{21} & G^{22} & G^{23} \\ G^{31} & G^{32} & G^{33} \end{vmatrix},$$

where $G^{ik}(x)$ stand for (algebraic) co-factors to the elements $g^{ik}(x)$. According to this, two tensor $\epsilon(x)$ and $\mu^{-1}(x)$ obey the following constraint

$$\epsilon(x)\mu^{-1}(x) = I.$$

Thus, the metric tensors with quasi-diagonal structure effectively describe media with following constitutive relations

$$D = -\epsilon_0\epsilon(x)\, E, \qquad B = \mu_0\mu(x)\, H, \qquad \mu(x) = -\epsilon(x)\,,$$

$$(\epsilon^{ik})(x) = \sqrt{-g(x)}\, g^{00}(x)\begin{vmatrix} g^{11}(x) & g^{12}(x) & g^{13}(x) \\ g^{21}(x) & g^{22}(x) & g^{23}(x) \\ g^{31}(x) & g^{32}(x) & g^{33}(x) \end{vmatrix}.$$

III. CONCLUSIONS

Application of the Riemannian geometry permits to simulate effective media which constitutive equations are determined by the metrical structure of the Riemann spaces. Because there are known numerous Riemannian geometries, the number of such effective media is enormous as well.

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ENERGY LEVELS OF ELECTRON IN CIRCULAR QUANTUM DOT IN THE PRESENCE OF SPIN-ORBIT INTERACTIONS

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I. INTRODUCTION

The motion of an electron in an inner layer of a double semiconductor heterostructure is usually treated as two-dimensional. In addition, the planar motion is also restricted if an electron is placed in quantum dot localized in a middle layer of heterostructure. The Rashba [1] and Dresselhaus [2] spin-orbit interactions are presented by the formulas $V_R = \alpha_R(\sigma_x p_y - \sigma_y p_x)/\hbar$ and $V_D = \alpha_D(\sigma_x p_x - \sigma_y p_y)/\hbar$, where σ_x and σ_y are the standard Pauli spin-matrices. The Rashba interaction strength can be controlled by an external electric field, and the Dresselhaus interaction strength can be varied by changing the width of quantum well along the z axis. In the general case the whole spin-orbit interaction has the form $V_R + V_D$. At the same time, the